Inviscid Flow Through Wide-Angle Diffuser with Actuator Disk

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Nomenclature

C_p = pressure coefficient, $(p-p_0)\ q_0$
C_{T_0} = disk loading coefficent based on freestream q, $\Delta p/q_0$
D = diffuser diameter
L = length of diffuser
p = static pressure
Δp = pressure drop across turbine or screen
q = velocity head
r = radial coordinate made dimensionless with L
R = diffuser radius made dimensionless with L
U_0 = dimensional free stream velocity
x = axial coordinate made dimensionless with L, measured from diffuser entrance
ψ = dimensionless stream function, ψ^*/U_0L^2
θ = diffuser half-angle

Subscripts
0 = freestream
2 = cross section just before turbine or screen, or at the diffuser entrance
3 = cross section just after turbine or screen
4 = cross section at diffuser exit

Superscript
* = dimensional quantity

RECENT developments have increased interest in diffuser-augmented wind turbines. The main development is the successful use of streamwise blowing to achieve very-wide-angle diffusers (up to 40 deg half-angle) without catastrophic separation that would have been expected at such large angles. Experiments and one dimensional theory for such diffusers are described in Refs. 1-3.

Kuchemann and Weber[4] have devised a shortcut method utilizing tabular results for carrying out axisymmetric potential flow calculations. Their example solution for a short $(L/D_2 = 1)$ 11-deg half-angle annular diffuser indicated a subatmospheric exit pressure $(C_p \simeq -0.3)$ and an increased flow through the diffuser—subsequently verified experimentally.

Since Kuchemann and Weber's work, a large number of investigators have applied the method of singularities to the computation of flow through annular wings.[5-7] In addition to using vortex distributions to represent the lifting effect (due to the Kutta condition) of the annulus and to represent the additional mass flow due to the propulsor, most of these analyses utilize source-sink and vortex combinations to represent the finite thickness of the shroud. The methods are suprisingly accurate, considering that they place the singularities on a cylinder of constant diameter.

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In this Note we describe a theoretical analysis of the inviscid flow past a diffuser-turbine combination, using the method of singularities. We have modified and extended the method of Ref. 4; the present approach, which does not constrain the vortices to lie on a cylinder, is expected to be more accurate than the calculations of Refs. 4-7. In our work, an appropriate distribution of ring vortices has been combined with a uniform flow along the axis to yield the desired flow. The boundary condition of zero normal velocity at the diffuser surface and at the wake bounding surface is used to determine the distribution of vorticity among the discrete vortices. The flow must also satisfy the Kutta condition of continuous fluid velocity at the diffuser trailing edge. Our computational scheme is an iterative one that considers successively a series of increasing disk loading coefficients, starting from zero (no turbine). As a first approximation, then, for each disk loading coefficient, the wake vortex configuration (strength and position) obtained for the previous (slightly lower) value of disk loading coefficient was used. A more detailed account of the work described in this Note is given in Ref. 8.

Analysis and Results

While potential flow analysis may at first seem useless for a situation in which the fluid loses total pressure while passing through a screen or turbine, the method can be used if the flow can be divided into two regions in each of which the total pressure is constant. In Ref. 4 this useful concept is fully explored. For our configuration, a potential flow solution is possible if we assume that the pressure drop across the screen or turbine is uniform over the entire cross section. For a turbine, the extent to which this assumption introduces an error will depend upon the turbine design.

Figure 1 shows the general features of our diffuser model in which discrete vortex rings are used to represent the diffuser and the wake bounding surface. The well-known equations for the vortex-induced velocities and stream function are given in detail in Ref. 8. An appropriate distribution of ring vortices has been combined with a uniform flow along the axis to yield the desired flow configuration. Specifically, in the present work, 10 equidistant ring vortices are used to approximate the diffuser surface, while 40 vortices are placed along the wake surface to simulate the turbine located at the upstream end of the diffuser. The distribution of vorticity along the diffuser surface and along the wake bounding surface must be such as to yield zero velocity normal to the surface. It must also satisfy the Kutta condition that the velocity be continuous at the diffuser trailing edge. The strength of the vortices and the radii of the wake vortices were adjusted iteratively until all these conditions were met.

Lines of constant stream function, i.e., streamlines, are plotted in Fig. 2 for a disk loading coefficient C_{T_0} of 0.8. We used $R_2 = 0.865$ and $\theta = 30$ deg in this plot in order to simulate closely some of the experimental work of Ref. 2. Only half of

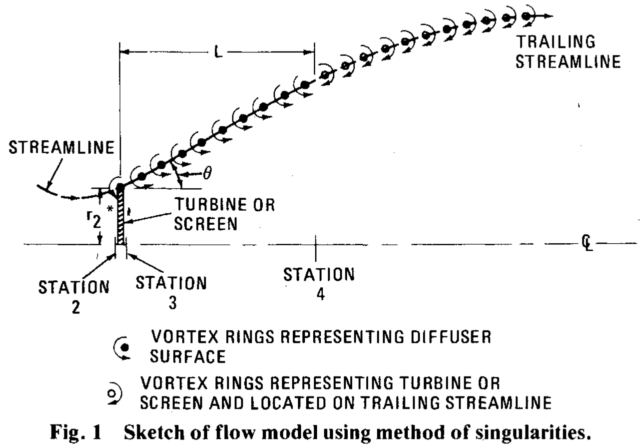

Fig. 1 Sketch of flow model using method of singularities.

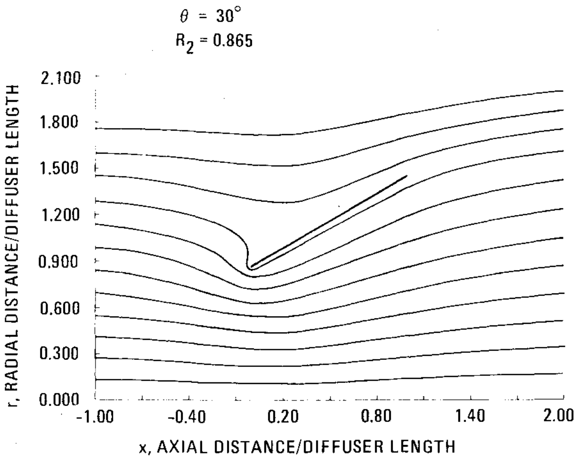

Fig. 2 Upper-half diffuser streamlines for $C_{T_0} = 0.8$.

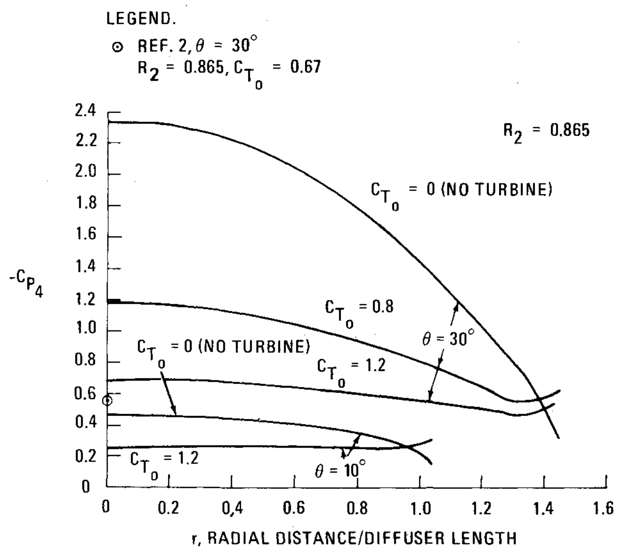

Fig. 3 Pressure coefficient profile in exit plane.

the flowfield is shown, with the abscissa axis ($\psi = 0$) representing the axis of symmetry of the flow. The diffuser wall itself is seen to extend between $x = 0$ and $x = 1.0$. The values of stream function ψ are not indicated in the illustration, but they have been chosen such that the streamlines are equidistant for uniform undisturbed flow (i.e., far upstream or far downstream). It is then easy to determine whether the flow is locally accelerated or retarded by observing whether the streamlines are more or less closely spaced, respectively. The effect of the diffuser in concentrating the flow through the turbine disk is evident in Fig. 2. This effect is explained simply by the circulation produced by the diffuser acting as an annular airfoil, with the inside surface of the diffuser being the high velocity, low pressure side of the airfoil.

In Fig. 3 is shown the variation of the exit plane pressure coefficient with radius. Consistent with the experimental findings of Ref. 2, the static pressure in the exit plane is seen to be subatmospheric (below freestream pressure). The addition of a flow resistance (turbine or screen) at the diffuser entrance has a strong flattening effect on the curves. As expected, the pressure reduction below atmospheric is much greater for the larger angle diffuser. The one experimental datum from Ref. 2 for $\theta = 30$ deg, $C_{T_0} = 0.67$, and $r = 0$ is seen to fall considerably below the value of $-C_{p_4}$ that one would estimate from interpolating the curves. This discrepancy is probably caused by the finite thickness of the wall and the boundary layer control flow and by viscous effects in the experiments that are not accounted for in the theory. The experiments utilized slots for injecting air from the external flow into the diffuser boundary layer to reduce separation, and no attempt was made to model these theoretically.

In conclusion, we have shown that the pressure reduction in the base region behind a wide-angle diffuser and the subsequent enhanced flow through the diffuser can be accounted for qualitatively by inviscid theory. The inviscid predictions are overly optimistic, however, indicating the need to account also for viscous effects for more accurate results.

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Some Effects of Unstable Resonators on Performance of CW Chemical Lasers

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THIS Note presents some computed results showing the effect of various cylindrical confocal unstable resonator configurations on the performance of a CW (continuous wave) chemical laser. These results have been obtained using the non-time-dependent LAMPPOST code[1,2] with the technique described in Ref. 3 for accelerating convergence of the solution. The LAMPPOST code has been developed by coupling Lockheed's Laser and Mixing Program[4,5] (known as the LAMP code) and a physical optics code for unstable resonators. The physical optics code developed by Altman uses the fast fourier transform techniques described by Salvi[6] and Phelps.[7] The effect of density variations on the index of refraction has not been included.

Figure 1 shows schematically the lasing region of a typical CW chemical laser operating with DF. Atomic fluorine and

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